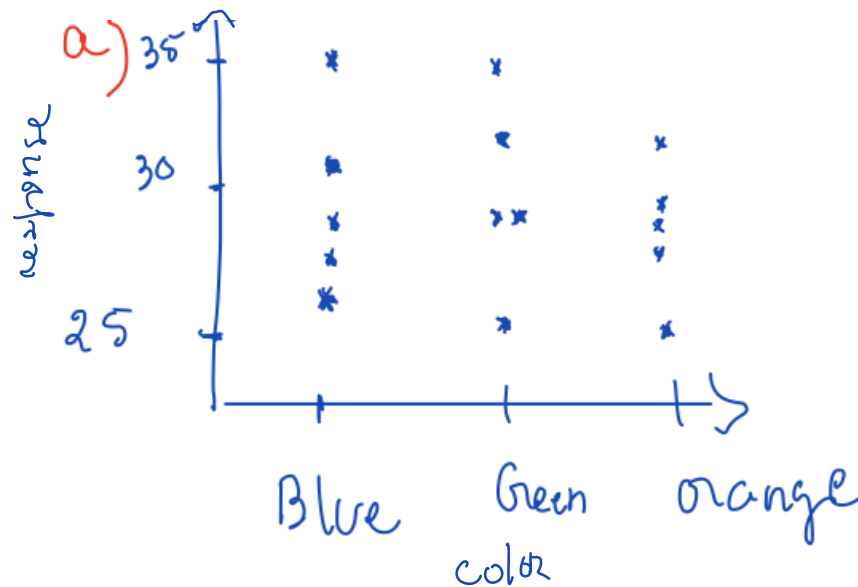


3. In an experiment to investigate the effect of color of paper (blue, green, orange) on response rates for questionnaires distributed by the "windshield method" in supermarket parking lots, 15 representative supermarket parking lots were chosen in a metropolitan area and each color was assigned at random of five of the lots. The response rates (in percentage) follow.

		1	2	3	4	5
1	Blue	28	26	31	27	35
2	Green	34	29	25	31	29
3	Orange	31	25	27	29	28

- (a) Prepare aligned dot plots of the data. Do the factor levels appear to be differ? Does the variability of the observations within each factor level appear to be approximately the same for all factor levels?



- Eventually orange has lower variability
- Levels of the factor (color) seem equal, eventually orange could be a little different.

(b) Obtain the analysis of variance table.

$n_1 = n_2 = n_3 = 5$ (Balanced Design)
 $N = 15$; $a = 3$

b) ANOVA TABLE

		1	2	3	4	5	$y_{i.}$
1	Blue	28	26	31	27	35	$y_{1.} = 147$
2	Green	34	29	25	31	29	$y_{2.} = 148$
3	Orange	31	25	27	29	28	$y_{3.} = 140$
							$y_{..} = 435$

$\sum_{i=1}^3 \sum_{j=1}^5 y_{ij}^2 = 12739 = 28^2 + 26^2 + \dots + 29^2 + 28^2$

$$SST = \sum_{i=1}^a \sum_{j=1}^{n_i} y_{ij}^2 - \frac{y_{..}^2}{N}$$

$$= 12739 - \frac{435^2}{15} = 124$$

$$SSTR = \sum_{i=1}^a \frac{y_{i.}^2}{n_i} - \frac{y_{..}^2}{N} = \frac{147^2 + 148^2 + 140^2}{5} - \frac{(435)^2}{15}$$

$$= 7.6$$

ANOVA table

Source of variation	SS	df	MS	F ₀
treatments	7.6	$(a-1) = 2$	$7.6/2 = 3.8$	$\frac{3.8}{9.7}$
Errors	116.4	$(N-a) = 12$	9.7	
total	124	$(N-1) = 14$	$\frac{124}{14}$	

$$SST = SSTR + SSE$$

$$\Rightarrow SSE = SST - SSTR$$

- (c) Conduct a test to determine whether or not the mean response rates for the three colors differ. Use level of significance $\alpha = 0.10$. State the alternatives, decision rule, and conclusion. What is the p-value of the test?

Model:

$$c) \quad Y_{ij} = \mu_i + \epsilon_{ij} = \mu + \tau_i + \epsilon_{ij} \quad \epsilon_{ij} \sim N(0, \sigma^2) \\ \text{iid}$$

$$E(Y_{ij}) = \mu_i = \mu + \tau_i$$

H_0 : mean of Y is equal for the three colours

$$\mu_1 = \mu_2 = \mu_3 \quad \text{vs } H_1: \exists(i,j): \mu_i \neq \mu_j$$

$$\Leftrightarrow H_0: \tau_1 = \tau_2 = \tau_3 = 0 \quad H_1: \exists i: \tau_i \neq 0$$

under H_0 , we have the test statistic:

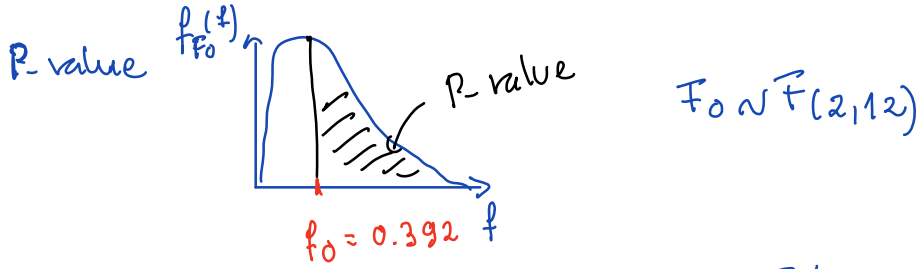
$$F_0 = \frac{MSTR}{MSE} \sim F_{(2,12)} \quad \alpha = 0.1$$

$$\text{ry } H_0 \text{ if } F_0 > c = F_{(2,12)}^{-1}(0.90) = 2.8068$$

$$c.r. =]2.8068; +\infty[$$

$$\text{obs. value: } f_0 = \frac{3.8}{9.7} = 0.392 \notin c.r.$$

As $f_0 \notin c.r.$, for $\alpha = 0.1$, it looks like that the colour paper has no influence in the mean responses rate



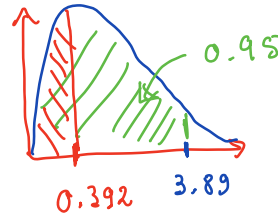
$$\begin{aligned}
 \text{P-value} &= P(F_0 \geq f_0) = P(F_0 \geq 0.392) = 1 - P(F_0 < 0.392) \\
 &= 1 - P(F_0 \leq 0.392) = 1 - \underbrace{F_{F_0}(0.392)}_{< 0.95} > 0.05
 \end{aligned}$$

Continuous
r. v.

table:

α	$n_2 \backslash n_1$	1	2
0.95	1	161.5	199.5
0.975		647.8	799.5
0.99		4052.2	4999.5
0.95	2	18.51	19.00
0.975		38.51	39.00
0.99		98.50	99.00
0.95	3	10.13	9.55
0.975		17.44	16.04
0.99		34.12	30.82
0.95	4	7.71	6.94
0.975		12.22	10.65
0.99		21.20	18.00
0.95	5	6.61	5.79
0.975		10.01	8.43
0.99		16.26	13.27
0.95	6	5.99	5.14
0.975		8.81	7.26
0.99		13.75	10.92
0.95	7	5.59	4.74
0.975		8.07	6.54
0.99		12.25	9.55
0.95	8	5.32	4.46
0.975		7.57	6.06
0.99		11.26	8.65
0.95	9	5.12	4.26
0.975		7.21	5.71
0.99		10.56	8.02
0.95	10	4.96	4.10
0.975		6.94	5.46
0.99		10.04	7.56
0.95	11	4.84	3.98
0.975		6.72	5.26
0.99		9.65	7.21
0.95	12	4.75	3.89
0.975		6.55	5.10
0.99		9.33	6.93

$$F_{F(2,12)}(3.89) = 0.95$$



$$\begin{aligned}
 \text{In R: } & pf(0.392, 2, 12) \\
 & F_{F(2,12)}(0.392) = 0.31595
 \end{aligned}$$

$$\text{P-value} = 1 - 0.31595 = 0.68405$$

ref $H_0 \forall \alpha \geq 0.68$

not ref $H_0 \forall \alpha < 0.68$

- (d) When informed of the findings, an executive said: "*See? I was right all along. We might as well print the questionnaires on plain white paper, which is cheaper*". Does this conclusion follow from the findings of the study? Discuss.

No, we did not test if the white paper would have the same effect than the blue, green or orange.